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MICHIGAN Department of Education

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A Parent's Guide to MATHEMATICS GRADE LEVEL CONTENT EXPECTATIONS

WHAT YOUR CHILD NEEDS TO KNOW BY THE END OF

SIXTH GRADE

MICHIGAN Department of Education
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www.michigan.gov/mde

Glossary Terms, continued

- **rational number –** any number that can be expressed as a quotient of 2 integers and can be in the form of fractions, mixed numbers, decimals, percents, integers, etc.
- **rectangular prism –** box shape; A prism is named by the shape of its bases which are connected by rectangular faces. A rectangular prism has rectangular bases; a triangular prism has triangular bases, etc.
- **scientific notation –** A method of writing or displaying numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10.
- **transversal –** A line that intersects parallel lines
- **terminating decimals –** decimals that end. For instance, $\frac{1}{2}$ can be written as a terminating decimal: 0.5. On the other hand $\frac{1}{3}$ cannot be written as a terminating decimal: it is 0.33333…with the 3's going on forever.

Bibliography

National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press. http://www.nap.edu/catalog/9822.html

For more information:

Michigan Department of Education Mathematics Page
www.michigan.gov/mathematics

Welcome to Our School!

This school year promises to be an exciting time for your child, filled with learning, discovery, and growth. It is also a time to share a new guide the Michigan Department of Education has developed for you. *A Parent's Guide to Grade Level Content Expectations* outlines the types of literacy and mathematics skills students should know and be able to do at the end of each grade.

Please feel free to share this guide with your family and friends. Use it when you talk with your child's teacher. Ask what *you* can do to support learning in the classroom and reinforce learning at home. You can find more ideas and tools to help you stay involved in your child's education at www.michigan.gov/mde.

Michigan Department of Education: GLCE Parent's Guide-6th Grade Math

A Parent's Guide to the Grade Level Content Expectations

Michigan Sets High Academic Standards – for ALL

This booklet is a part of Michigan's Mathematics and English Language Arts Grade Level Content Expectations (GLCE). It is just one in a series of tools available for schools and families. The Michigan Department of Education (MDE) provides similar booklets for families of children in kindergarten through eighth grade.

Teacher versions of the Grade Level Content Expectations are finished for grades kindergarten through eight. They state in clear and measurable terms what students in each grade are expected to know and be able to do. They also guide the design of the state's grade level MEAP tests required in the No Child Left Behind Act (NCLB) legislation.

Educators and classroom teachers from Michigan school districts have been involved in the development and/or review of Michigan's GLCE. The expectations were designed to ensure that students receive seamless instruction, from one grade to the next, leaving no gaps in any child's education. More importantly, they set high expectations in literacy and mathematics so we can better prepare all K-12 students for the challenges they will face in a global 21st century.

To learn more about the Michigan Curriculum Framework, visit www.michigan.gov/mde and click on **"K-12 Curriculum"**.

GLOSSARY TERMS

- **absolute value –** the numerical value of a real number without regard to its sign
- **algorithm –** a prescribed procedure for solving a problem, including any mathematical operation
- **coefficient –** indicates how many variables are used in an expression or equation, i.e. 3x or 5y (3 and 5 are coefficients)
- **congruent –** same size and/or shape
- **coordinate plane –** a plane spanned by the x-axis and y-axis in which the coordinates of a point are its distances from two intersecting perpendicular axes.
- **function –** a rule that assigns one output value to each input value. For example f(x) =3+x means for every x that is inputted, 3 is added to it to get the output. If 2 is put into the function than the output is 3+2 or 5.
- **geometry –** the area of mathematics that involves shape, size, space, position, direction and movement. It describes and classifies the physical world in which we live.
- **integers –** positive and negative whole numbers and 0
- **linear –** forming a straight line
- **product –** the number obtained when multiplying numbers
- **quotient –** the number obtained when dividing one number by another Example: 15÷3= 5, 5 is the quotient.
- **ratios –** the relation between two quantities expressed as the quotient of one divided by the other

Michigan Department of Education: GLCE Parent's Guide-6th Grade Math

DATA AND PROBABILITY

Understand the concept of probability and solve problems

- Express probabilities as fractions, decimals, or percentages between 0 and 1.
  Example: When a coin is flipped there is 1 out of 2 chances it will be heads. This probability can be written as $\frac{1}{2}$, 0.50 or 50%.

- Know that 0 probability means an event will not occur and that probability 1 means an event will occur.
  Example: There is 0 probability that if a child reaches into a drawer with 8 white socks (and only white socks), the child will pull out a red sock (0 out of 8, $\frac{0}{8}$, 0.0 or 0%). There is a probability of 1 that the child will pull out a white sock (8 out of 8, $\frac{8}{8}$, 1.0 or 100%)
  .

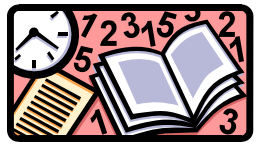
- Compute probabilities of events from simple experiments with equally likely outcomes by listing all possibilities and finding the fraction that meets given conditions.
  Examples: tossing dice, flipping coins, spinning spinners

Mathematics is the science of patterns and relationships. It is the language and logic of our technological world. Mathematical power is the ability to explore, to imagine, to reason logically and to use a variety of mathematical methods to solve problems; all important tools for children's futures. A mathematically powerful person should be able to:

- reason mathematically,
- communicate mathematically,
- solve problems using mathematics, and
- make connections within mathematics and between mathematics and other fields.

Michigan's **Mathematics Grade Level Content Expectations** (GLCE) are organized into five strands:

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data and Probability



In the sixth grade, students complete their work with Number and Operations. Students progress from whole numbers and fractions to integers and rational numbers. This sets the foundation for algebra, which begins in grade six. Students use variables, write simple equations and expressions, and graph linear relationships. In geometry, students continue to explore shapes and their properties.

Purpose of this Parent Document

This document is intended to give parents an overview of the Grade Level Content Expectations. Its intent is to provide an explanation of *why* students are doing particular homework assignments. It is a tool to facilitate communication between parents and teachers as parents become partners in their student's education.

Glossary Terms

Words that have asterisks (*) are defined in the Glossary located in the back of this booklet.

Number and Operations

Multiply and Divide Fractions

Students extend their knowledge of multiplication & division of whole numbers to the multiplication & division of fractions. Although division skills are important, understanding the concept of division leads to the deeper understanding of mathematics necessary in today's world. "Learning with understanding is more powerful than simply memorizing because the organization improves retention, promotes fluency, and facilitates learning related material" (National Research Council, pg. 118.) The mathematical "habits of mind" that students develop as they extend their understanding of multiplication and division of whole numbers to all rational numbers are laying the framework for the algebra they are introduced to in 6th grade and will delve into more deeply through the 7th and 8th grades.

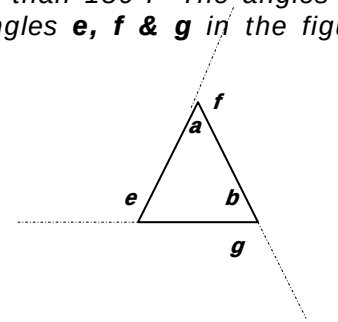
- ❑ Understand division of fractions as the opposite of multiplication.
- ❑ Write a number sentence representing a situation involving dividing fractions.
- ❑ Solve for the unknown in equations.
  Example: ¼ ÷ ___ = 1 ¾
- ❑ Multiply and divide any two fractions, including mixed numbers.
  By the end of 6th grade, students are expected to have developed algorithms (procedures) that allow them to compute efficiently and accurately with fractions.

Find equivalent ratios

- ❑ Find equivalent ratios* by scaling up or scaling down.
  $\frac{1}{2}$ *increased by a scale factor of 2* $= \frac{2}{4}$ $(\frac{1}{2} x \frac{2}{2})$; $\frac{4}{12}$ *decreased by a scale factor of 4* $= \frac{1}{3}$ $(\frac{4}{12} \div \frac{4}{4})$.

Understand and apply basic properties of lines, angles and triangles, continued

- ❑ Know that the sum of the exterior angles of a convex polygon is 360°.
  *A convex polygon is a shape with 3 or more sides whose interior angles are less than 180°. The angles formed by the extended sides (angles **e, f & g** in the figure below) add up to 360°.*



Understand the concept of congruence and basic transformations

Before 6th grade, students have flipped (reflected), turned (rotated) and slid (translated) geometric figures. They now need to know the formal terms for these actions and realize that they only change the position of polygons and not their size or shape.

- ❑ Understand that for polygons, congruence means corresponding sides and corresponding angles have equal measures.
- ❑ Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.
- ❑ Understand and use simple compositions of basic rigid transformations.
  Example: A translation followed by a reflection.

Construct geometric shapes

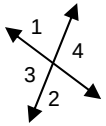
- ❑ Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors; justify informally.

GEOMETRY

Understand and apply basic properties of lines, angles and triangles including:

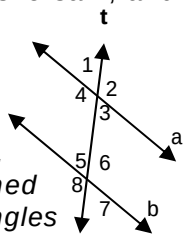
- Triangle inequality: *The length of the third side of a triangle is always less than the sum of the other two sides and greater than their difference.*

- Relationships of vertical angles, complementary angles, and supplementary angles



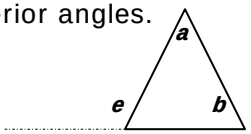
Vertical: Angles 1 & 2 and 3 & 4 are vertical angles because they are opposite each other.
Supplementary: Pair of angles whose measures add up to 180°: angles 1&3 are supplementary as are angles 2&3, 1&4, & 2&4.
Complementary: Pair of angles whose measures add up to 90°.

- Congruence* of corresponding and alternate interior angles when parallel lines are cut by a transversal*, and that such congruencies imply parallel lines



Angles are congruent (≅), if they have the same measure.
*In this example, lines **a** and **b** are parallel and they are cut by line **t**, the transversal*.*
***Corresponding** angles are the angles formed by lines **a** and **t** that "correspond" to the angles formed by lines **b** and **t**. For example, angle 1 corresponds to angle 5 so angle 1 ≅ angle 5.*
***Alternate interior** angles in this figure include angles 5 & 3; so, angle 5≅3.*

- Locate interior and exterior angles of any triangle, and use the property that an exterior angle of a triangle is equal to the sum of the remote (opposite) interior angles.



*Angle **e** is equal in measure to angle **a** + angle **b**.*

Represent rational numbers as fractions or decimals

A rational number is any number that can be expressed as a quotient of any two integers*(non-zero denominators) and can be in the form of fractions, mixed numbers, decimals, percents, integers*, etc. Sixth graders' previous work with fractions and decimals begins to develop into an understanding of all rational numbers. They begin to understand that the same number can be represented in different ways and they recognize that the form of the number they use depends on the context in which it is being used.*

- Represent rational numbers as fractions or terminating decimals* when possible, and translate between these representations.

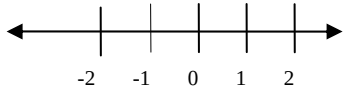
$\frac{1}{8}$=.125; $\frac{8}{3}$ is a rational number because it can be represented as a fraction even though it cannot be represented by a terminal decimal ($\frac{8}{3}=2.6\overline{6}$).

- Understand that a fraction or a negative fraction is a quotient of two integers* (non-zero denominators) and that rational numbers are quotients of integers, therefore a rational number is either a fraction or a negative fraction.

Example: $-\frac{4}{2}=-4\div 2$. $\frac{8}{3}$ is $8\div 3$; $\frac{8}{3}$ and $-\frac{4}{2}$ are both rational numbers.

Understand rational numbers and their location on the number line

- Understand that 0 is an integer that is neither negative nor positive.
- Order rational numbers and place them on the number line.
- Know that numbers and their negatives add up to 0, and are on opposite sides at an equal distance from 0 on the number line.



$-2+2=0$

Understand rational numbers and their location on the number line, continued

- ❑ Know that the absolute value* of a number is the value of the number, ignoring the sign, or is the distance of the number from 0.
  The absolute value of a number is indicated with | |. For example: the absolute value of 3 is written as $|3|$ and the absolute value of -3 is also written as $|3|$.

Compute with integers and rational numbers

Students' understanding of the relationship between addition and subtraction becomes more sophisticated as they recognize that integer subtraction is really the opposite of integer addition (adding 3 goes the opposite way on the number line than subtracting 3; adding -3 is the same as subtracting 3). Furthermore, they learn the rules of the signs (negative times a negative is a positive; negative times a positive is a negative) by using the relationship between multiplication and repeated addition; and between multiplication and division. (-3+-3=-6 ⇒-3 x 2=-6 ⇒ -6÷2=-3).

- ❑ Understand integer subtraction as the opposite of integer addition.
- ❑ Add, subtract, multiply, and divide integers between -10 and 10.
- ❑ Add, subtract, multiply, and divide positive rational numbers fluently.
  (This includes integers, fractions and decimals.)
- ❑ Estimate the answers to calculations involving operations with rational numbers.

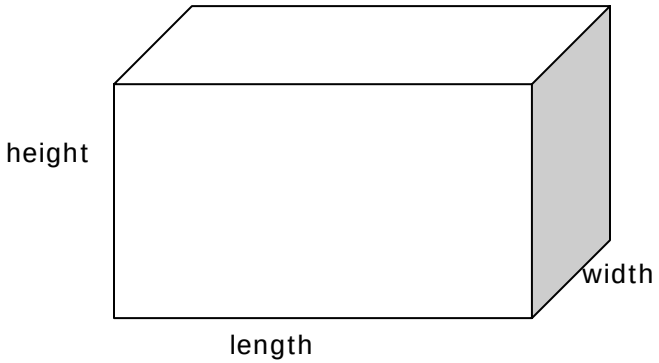
MEASUREMENT

Convert within measurement systems

- ❑ Convert between basic units of measurement within a single measurement system.
  Example: Convert square inches to square feet; minutes to hours, etc.

Find volume and surface area

- ❑ Draw patterns (of faces*) for a cube* and rectangular prism* that, when cut, will cover the solid exactly (nets).
  Students need to visualize what a cube looks like if it is opened flat.
- ❑ Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides using formulas.
  Example:
  Volume formula: length x width x height
  Surface area: the sum of the surface area of all 6 faces



Rectangular Prism

Represent linear functions* using tables, equations, and graphs, continued

- ❑ Represent simple relationships between quantities using verbal descriptions, formulas or equations, tables, and graphs.
  Example: Distance-time relationships such as miles per hour; conversions such as feet to inches

Solve equations

- ❑ Relate simple linear equations with integer coefficients* to particular contexts, and solve.
  Example: A triangle has three sides and if an unknown number of triangles (x) had 12 sides this could be represented by the linear equation 3x=12.
- ❑ Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution.
  Example: x+8=15
  *x+8**-8**=15**-8***
  x=7;
- ❑ Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same solutions.
  Example: x+8=15
  ***2**(x+8) = **2** (15)*
  2x+16=30
  2x=14
  x=7
- ❑ Solve equations of the form ax + b = c, Example:
  5x + 6 = 21, by hand for positive integer coefficients* less than 20, using calculators otherwise, and interpret the results.
  A coefficient indicates how many x's are used in the equation.

Solve decimal, percentage, and rational number problems

- ❑ Calculate part of a number given the percentage and the number.

- ❑ Solve word problems involving percentages related to sales taxes and tips.

Example: A CD is priced at $15.00. What is the total cost with 6% sales tax?
Students first need to determine the amount of the tax by finding 6% of 15. They can do this by multiplying 15 by $\frac{6}{100}$ *or by .06 to come up with $.90. This makes the total cost of the CD 15+.90=$15.90*

- ❑ Solve problems that use the four operations with appropriate decimal numbers.

Use Exponents

An exponent is a superscript that shows how many times a number or expression is to be multiplied by itself: 4^3 = 4x4x4=62; 10^2 = 10x10 (10 to the second power). Exponents are often used to make very large or very small numbers (in the case of negative exponents) easier to use. As an example, 1,000,000 can be written as 10^6. Since scientists often work with very big or very small numbers they often take advantage of exponents by putting numbers in "scientific notation". As an example, the distance from the Earth to the Sun is approximately 150,000,000 km. Converted to scientific notation this distance is now 1.5 x 10^8km which means that 1.5 is multiplied by 10, 8 times.

- ❑ Understand and use integer exponents.

- ❑ Express integer exponents in scientific notation.

ALGEBRA

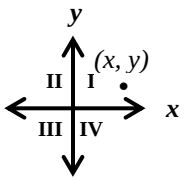
Students begin their formal study of algebra in sixth grade; however, they have already been exposed to many of these ideas during their work with numbers. Algebra involves recognizing relationships between variables and using those relationships to solve problems. These relationships are found by using tables, equations or graphs. Therefore, 6th graders need to understand how to read and make graphs; write, simplify, and solve equations and expressions; and translate between all three of these representations.

Calculate rates

- ❑ Solve problems involving rates including speed.
  Example: If a car is going 50 mph, how far will it go in 3½ hours?

Understand the coordinate plane

- ❑ Plot ordered pairs of integers and use ordered pairs of integers to identify points in all four quadrants of the coordinate plane.



Example: (x,y) is a point in Quadrant I of the coordinate plane.

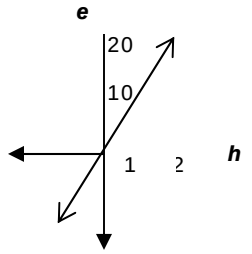
Use variables, write expressions and equations, and combine like terms

- ❑ Use letters with units to represent quantities in a variety of contexts; *Example: y lbs., k minutes, x cookies.*
- ❑ Distinguish between an algebraic expression* and an equation.
- ❑ Use standard conventions for writing algebraic expressions.
  Example: 2x+1 means "two times x, plus 1" and 2(x+1) means "two times the quantity (x+1)."
- ❑ Represent information given in words using algebraic expressions and equations.
- ❑ Simplify expressions of the first degree by combining like terms, and evaluate using specific values.
  Example: Simplify 3x+5x⇒ 8x
  Simplify a+2a+3b: 3a+3b⇒3(a+b)
  Evaluate 8x if x is 5: 8(5) =40
  Evaluate 3(a+b) if a is 4 and b is 5: 3(4+5) =3(9) =27.

Represent linear functions* using tables, equations, and graphs

Understanding functions is a foundation of algebra. A function describes the relationship between an input and an output. For example, the amount of money earned can be a function of the number of hours worked. This relationship can be represented with a:

graph



table

Hours worked (h)	*Earnings (e)*
1	*$10*
2	*$20*

Equation *f(h)=10h*
(the "function" of h is 10h)

- ❑ Understand that graphs and tables can suggest relationships between quantities.
- ❑ Graph and write equations for linear functions of the form y=mx, and solve related problems.