

9/15/15

LESSON 3: NUMBERS IN EXPONENTIAL FORM RAISED TO A POWER

Lesson Objective: You will learn how to take powers of powers. You will learn that when a product is raised to a power, each factor of the product is raised to that power.

Classwork

For any number x and any positive integers m and n ,

$$(x^m)^n = x^{mn}$$

because

$$\begin{aligned} (x^m)^n &= \underbrace{(x \cdot x \cdots x)}_{m \text{ times}}^n \\ &= \underbrace{(x \cdot x \cdots x)}_{m \text{ times}} \times \cdots \times \underbrace{(x \cdot x \cdots x)}_{m \text{ times}} \quad (n \text{ times}) \\ &= x^{mn} \end{aligned}$$

Exercise 1

$$(15^3)^9 = (15 \cdot 15 \cdot 15) \cdot (15 \cdot 15 \cdot 15) \cdot \dots \cdot (15 \cdot 15 \cdot 15) \quad \text{9 times}$$

$$15^{27}$$

Exercise 3

$$(3.4^{17})^4 = (3.4)^{68}$$

Exercise 2

$$((-2)^5)^8 = (-2)^{40}$$

Exercise 4

Let s be a number.

$$(s^{17})^4 = s^{68}$$

Exercise 5

Sarah wrote that $(3^5)^7 = 3^{12}$. Correct her mistake. Write an exponential expression using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

$$3^{35} \quad \text{and} \quad 3^5 \cdot 3^7 = 3^{12}$$

Exercise 6

A number y satisfies $y^4 - 256 = 0$. What equation does the number $x = y^4$ satisfy?

Since $x = y^4$

$$(x)^6 = (y^4)^6 = y^{24}$$

$$x^6 - 256 = 0$$

For any numbers x and y , and positive integer n ,

$$(xy)^n = x^n y^n$$

because

$$\begin{aligned} (xy)^n &= \underbrace{(xy) \cdots (xy)}_{n \text{ times}} \\ &= \underbrace{(x \cdot x \cdots x)}_{n \text{ times}} \cdot \underbrace{(y \cdot y \cdots y)}_{n \text{ times}} \\ &= x^n y^n \end{aligned}$$

Exercise 7

$$(11 \times 4)^9 = 11^9 \times 4^9$$

Exercise 10

Let x be a number.

$$(5x)^7 = 5^7 \cdot x^7$$

Exercise 8

$$(3^2 \times 7^4)^5 = 3^{10} \times 7^{20}$$

Exercise 11

Let x and y be numbers.

$$(5xy^2)^7 = 5^7 \cdot x^7 \cdot y^{14}$$

Exercise 9

Let a , b , and c be numbers.

$$(3^2 a^4)^5 = 3^{10} \cdot a^{20}$$

Exercise 12

Let a , b , and c be numbers.

$$(a^2 bc^3)^4 = a^8 \cdot b^4 \cdot c^{12}$$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n ?

PROBLEM SET

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$(2 \cdot 3 \cdot 4)^4 = (2 \cdot 3 \cdot 4)(2 \cdot 3 \cdot 4)(2 \cdot 3 \cdot 4)(2 \cdot 3 \cdot 4)$
 $(2 \cdot 2 \cdot 2 \cdot 2)(3 \cdot 3 \cdot 3 \cdot 3)(4 \cdot 4 \cdot 4 \cdot 4)$
 $2^4 \cdot 3^4 \cdot 4^4$

1. Show (prove) in detail why $(2 \cdot 3 \cdot 4)^4 = 2^4 3^4 4^4$.

2. Show (prove) in detail why $(xyz)^4 = x^4 y^4 z^4$ for any numbers x, y, z .

3. Show (prove) in detail why $(xyz)^n = x^n y^n z^n$ for any numbers x, y, z , and for any positive integer n . $n=3$